THE PROBLEM OF WATER POLLUTION: OXYGEN-SAG MODEL

N. V. RAMANA MURTY
Dept. of Mathematics, Andhra Loyola College, Vijayawda-520008
Andhra Pradesh, INDIA

ABSTRACT

To live happily in this universe, every lively body requires fresh air and fresh water which was available till two centuries ago. After that, many developments came in the technology of all countries and ultimately the fresh environment of healthy living has been damaged almost completely. Many factors affect the environment among which 'Water Pollution' is a big problem. In many cities, big rivers are polluted by joining factory outlets to leave its excrements into the river which causes water pollution. Against this background, this paper makes an attempt to study oxygen-sag model problem of rivers. This problem was first raised by Streeter and Phelps in1925. This model predicts the concentration and deficit of oxygen caused by a discharge of sewage containing Biochemical Oxygen Demand (BOD) which consumes oxygen.

Keywords: Oxygen-sag problem, Biochemical Oxygen Demand, Water Pollution

Introduction:

Oxygen-sag problem is a problem of calculating the concentration and deficit of oxygen in a river caused by a discharge of sewage containing BOD. This problem involves several parameters like process of de-oxygenation and re-oxygenation as a function of time, under ideal flowing conditions of the river. The concentration of the oxygen levels in a river is disturbed by de-oxygenation because of sewage containing BOD. Since sewage BOD is biodegraded aerobically by natural bacteria, it also one of the causes in disturbing the oxygen concentration in a flowing river. Therefore, this is a common problem for ecology and environment. To calculate all these things, there is a need to consider the application of differential equations.

Main Results:

The rate of change of ultimate BOD in a river as a function of actual time of flow can be represented by a first order differential equation $\frac{dB}{dt} = -kB \rightarrow (1)$, where B is the ultimate BOD in mg/L. Solving this equation (1), it gives $B_t = B_b e^{-kt} \rightarrow (2)$, where B_t, B_b are ultimate BOD at time t and at dilution point b respectively. It is known that BOD reduction is related to oxygen consumption, the resulting change of oxygen concentration due to biochemical reaction is represented by the differential equation $\frac{dC_{de}}{dt} = \frac{dB}{dt}$, where C_{de} is concentration of de-oxygenation. By using (1) and (2), the above equation becomes $\frac{dC_{de}}{dt} = -kB \Rightarrow \frac{dC_{de}}{dt} = -kB_b e^{-kt} \rightarrow (3)$

Similarly, the rate of re-oxygenation is given by the differential equation $\frac{dC_{re}}{dt} = r(C_s - C) \rightarrow (4)$, where, C_{re} is concentration of re-oxygenation, r is re-oxygenation constant, C_s is oxygen saturation concentration in mg/L, C is Oxygen concentration in mg/L.

Therefore, the resulting amount of oxygen in the river is given by the differential equation

$\frac{dC}{dt} = \frac{dC_{re}}{dt} + \frac{dC_{de}}{dt}$. By using (3) and (4), it gives $\frac{dC}{dt} = r(C_s - C) - kB_b e^{-kt} \rightarrow (5)$.

Definition: The oxygen deficit is denoted by D and is defined as oxygen saturation concentration and oxygen concentration. That is, $D = C_s - C$.

The rate of oxygen deficit in the polluted river is given by the differential equation $\frac{dD}{dt} = kB_b e^{-kt} - rD \rightarrow (6)$. Solving this equation, the solution gives the deficit of oxygen as function of time. That is, $D_t = \frac{kB_b}{(r-k)}\left(e^{-kt} - e^{-rt}\right) + D_b e^{-rt} \rightarrow (7)$.

The algebraic simplification of all these equations gives the ultimate BOD B_b at the dilution point b, the Oxygen concentration C_b at the dilution point b, and the deficit of Oxygen D_b at the dilution point b, are given by

$B_b = \frac{Q_r B_r + Q_{sw} B_{sw}}{Q_r + Q_{sw}} \rightarrow (8)$,

$C_b = \frac{Q_r C_r + Q_{sw} C_{sw}}{Q_r + Q_{sw}} \rightarrow (9)$, and

$D_b = C_s - C_b \rightarrow (10)$, respectively, where $Q_r =$ the flow rate of river;

$B_r =$ ultimate BOD of river;

$C_r =$ oxygen concentration along the river;
$Q_{sw} =$ flow rate of sewage;
$B_{sw} =$ ultimate BOD of sewage;
$C_{sw} =$ oxygen concentration in sewage;
$C_s =$ oxygen saturation concentration; and
$C_b =$ oxygen concentration at dilution point;

Conclusion:

Since rivers are getting polluted because of this Oxygen-sag problem, the Governments of all countries must formulate a policy that factory excrements should not be discharged into rivers. If such a legal policy is made by the Government, water of every river will be afresh and the drinking water problem will be solved to great extent.

References:

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